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CELESTIAL MECHANICS PROBLEMS OF SPACE FLIGHT: PART II.

By K. Schuette

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CELESTIAL MECHANICS PROBLEMS OF SPACE FLIGHT. PART II.

[Following is the translation of "Himmelmechanische Probleme der Raumfahrt" (English version above) by K. Schuette in Flugkoerper, Vol. I, January 1960, pages 10 - 15.]

II. The First Calculations of Flight Paths to the Moon and around the Moon.

Some inferences from the problem of a rocket path to the moon

The calculations of E. Stromgren and his coworkers have resulted in the successful investigation of at least one case of the extended "probleme restreint" relating to periodic orbits. The many hundreds of orbital calculations, which they have made in the course of a generation's work, largely made possible by the financial support of the Carlsberg Foundation, Copenhagen, constitute a valuable pioneering effort.

It will be necessary to continue and extend this work, if we are to compute with certainty and master the paths to be followed by a space ship on its way to the moon and beyond.

Two circumstances, which are of decisive importance in finding such paths, deserve special emphasis.

Firstly, with the aid of electronic computers it is now possible to complete the calculations themselves more than one thousand times as fast as before. Without this novel assistance it would scarcely be possible for flight calculations to keep pace with technical developments. However, it must not be forgotten that the preliminary work involved in programming, which, it is true, need be performed only once, itself requires a considerable amount of time.

Secondly, within the foreseeable future it will be necessary to go beyond the phase of socalled passive flight and develop active flight paths.

Whereas in passive flight the path is determined by the starting data, in active flight the speed can be varied by means of an additional impulse, triggered during the course of the flight. This would be practically the same as starting with another initial velocity. Or in other words: if a kind of Hill boundary curve (given by the initial conditions) exists for the original path, in active flight it will be possible to go beyond it by applying an additional impulse. Here lies the essential difference with respect to the paths followed by all the natural celestial bodies. As far as the author knows, the Hill boundary curves for four bodies with the mass relationships given by the solar system (sun, earth, moon, rocket) have not yet been investigated.

The particular conditions for a rocket flight to the moon

When we come to make practical use of our knowledge of the possibilities of motion in the planetary system, we realize that the mass of the sun is very much the dominating factor. Even Jupiter, the giant planet, has a mass only 1/1,047th of that of the sun. Thus, many problems of motion can be treated as perturbation problems.

In our present considerations we may first disregard the other planets because of their relatively small masses and great distances and, as far as flight paths to and around the moon are concerned, restrict ourselves to the four bodies - sun, earth, moon, rocket. If we put the mass of the earth equal to 1 and take the average distance between earth and moon (384,403 km) as the unit of distance, we shall have the following mass ratios and distances:

Masses	mean distances
$m_{earth} = 1$	earth-moon = 1
$m_{sun} = 333,434$	sun-moon = 389.4
$m_{moon} = 1:81.31$	earth-sun = 389.4
m_{rocket} = (practically zero)	

Doubtless we are faced with a true four-body problem. However, the conditions are complicated by the fact that on launching and in the vicinity of the earth the attraction of the earth predominates. Nevertheless, the abaric point between earth and sun is only about two-thirds of the lunar distance away. From this point on the

attraction of the sun predominates; naturally, it makes itself felt even before this but in the vicinity of the earth is completely overshadowed by the attraction of the earth itself.

Thus, a space ship which goes beyond two-thirds of the lunar distance is subject mainly to the attraction of the sun. Now if the moon lies in the direction of motion of the space ship so that the latter approaches the moon, then the abaric point between earth and moon lies at nine-tenths of the lunar distance or about 23 lunar radii away from the moon. If this point is passed, the space ship arrives in a region where the attraction of the moon predominates. At the abaric point between earth and moon the accelerations due to the mutual forces of attraction act in opposite directions and are equal in magnitude; they are still only 0.332 cm/sec^2. Fig. 10 illustrates the relationships schematically in a rotating coordinate system. At the abaric point N a quite small impulse is sufficient to take the space ship around the moon or else to cause it to impact upon the latter.

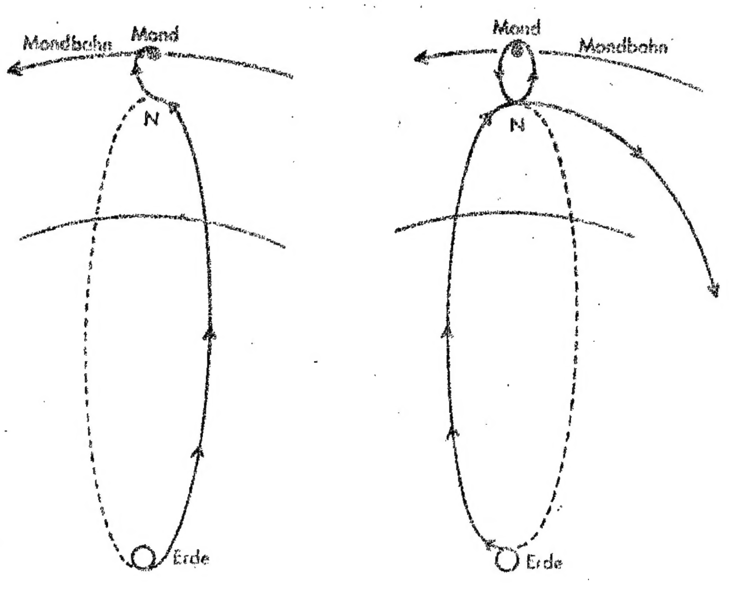

Fig. 10: Problems of a flight path to the moon (schematic) (From: K.Schuette, Die Weltraumfahrt hat begonnen (Space travel has begun), Herder-Taschenbuch, No. 11, 2nd ed. Freiburg, 1958, p. 160. (Mond = moon; Erde = earth; Mondbahn = orbit of moon).

Finally, there is yet another abaric point, between the sun and the moon. At the earth-sun distance the acceleration due to the attraction of the sun is still about 0.6 cm/sec^2; at about 16.5 lunar radii from its mid-point that due to the moon has also decreased to this amount. It follows that a circle, which contains the abaric point between sun and moon, lies somewhat closer to the moon than the corresponding one for earth and moon. More precisely, then, a space ship, which has passed the abaric point between earth and moon at a distance of 23 lunar radii, remains in the region dominated by the attraction of the sun, until it has approached within 16.5 lunar radii of the center of the moon.

These relationships, merely hinted at here, suggest why the theory of lunar motion is the most difficult problem in celestial mechanics we know.

Orbital and escape velocities at various altitudes above the earth's surface and the launching velocity required to travel lunar distances

If R is the radius of the earth and g_o the acceleration due to gravity at the earth's surface, then the orbital velocity v_o at the altitude H km above the surface of the earth is given by the good approximation formula:

$$v_0 = R\sqrt{\frac{g_0}{R+H}}. \qquad (28)$$

The corresponding escape velocity v_∞ is then found from the energy integral to be:

$$v_\infty = \sqrt{2}\cdot v_0. \qquad (29)$$

Thus, the decrease in gravity with height, but not the frictional resistance of the atmosphere, is taken into account.

Taking R = 6,378.24 km and g_o = 9.80665 m/sec^2 we get the following values for v_o and v_∞ at altitudes of from 0 to 500 km:

H_{km}	v_c km/sec	v_∞ km/sec
0	7,909	11,183
50	7,878	11,141
100	7,847	11,098
150	7,817	11,056
200	7,788	11,014
250	7,758	10,972
300	7,729	10,931
350	7,700	10,890
400	7,672	10,850
450	7,644	10,810
500	7,616	10,770

It is evident that in the lowest layers the orbital velocity decreases by 31 to 28 m/sec for every 50 km increase in altitude, while the corresponding figures for the escape velocity are 44 to 40 m/sec.

Between circular orbit and parabola (escape velocity) lie all the possible elliptical paths. The nature of the path is determined solely by the magnitude and direction of the initial velocity (at burn-out), that is by the velocity vector. However, the attainable distance is determined only by the magnitude of the velocity, not by the direction (see energy integral), whereas the direction of the velocity vector is linked with the position and form of the path. From the astronomical point of view, the problem of determining orbits from the velocity vector has so far been of no interest, as this vector is generally unknown where newly discovered celestial bodies are concerned. As far as the approach of world space travel is concerned, this problem of the determination of an orbit from the velocity vector was first solved by the author in 1953/54 for the case of motion in a plane [24].

If we now inquire how great the speed must be to achieve lunar distances in passive flight within the two-body problem, we find that it must be very close to the escape velocity. On increasing the launching velocity to, say, 10.0 km/sec a maximum altitude of about 4 earth radii is attained. 11 km/sec would take us to an altitude of somewhat more than 28 earth radii, that is just half the distance to the moon. If the launching velocity is increased beyond this point in 10 m/sec steps, then, theoretically, we get the following relationships, assuming that at burn-out a turn is made into an elliptical orbit at right angles to the radius of the earth:

Characteristics of an elliptical orbit on increasing the launching velocity in the immediate vicinity of of the escape velocity (in the two-body problem) [25]

Launching velocity (at burn-out) km/sec	Eccentricity of orbit	Attained at apogee: Max. distance in earth radii	Attained at apogee: Min. orbital velocity km/sec	Orbital period: Days	Orbital period: Hrs.
11,02	0,939	31,77	0,347	3	21,6
11,03	0,942	33,70	0,327	4	6,0
11,04	0,946	36,05	0,306	4	16,1
11,05	0,950	38,64	0,286	5	4,1
11,06	0,953	41,62	0,266	5	18,3
11,07	0,957	45,09	0,246	6	11,5
11,08	0,960	49,18	0,225	7	8,7
11,09	0,964	54,07	0,205	8	11,2
11,10	0,967	60,02	0,185	9	21,0
11,11	0,971	76,88	0,165	11	17,5

Here the actual height of the beginning of the orbit at burn-out is not taken into account. Furthermore, the initial data for R and g_o are not quite the same as in the preceding table. What is important, however, is the recognition that on approaching the escape velocity even a slight increase in the launching velocity can have a quite considerable effect on the maximum distance attained at apogee. It is also possible to imagine that an angular error in the starting velocity might likewise be of the greatest importance. The escape velocity of 11.185 km/sec, given above, is probably sufficient to leave the earth but it is not sufficient, if the intention is to leave the solar system. At the earth's distance from the sun its orbital velocity about the sun is 29.766 km/sec; this is precisely the mean velocity of the earth itself in its very nearly circular annual journey around the sun. Thus, the escape velocity required to leave the solar system from the position of the earth, but without taking into account its own attraction, would be $\sqrt{2} \times 29.766$ km/sec = 42.095 km/sec.

However, it would be false to assume that if the launching were in the direction of the tangent to the earth's orbit and in the same sense as the motion of the earth, only 42.095 - 29.766 = 12.33 km/sec would be required to leave the solar system. The figure must be further increased, as in any event it is necessary to overcome the earth's gravitational field.

The importance and scope of numerical flight calculations

Our consideration of the three- and four-body problem was deliberately made somewhat detailed in order to establish clearly and unmistakably that a generally valid analytical solution for a flight path to the moon does not exist. For many purposes it may suffice to regard determination of the flight path as a three-body problem; however, it would be surer and in some cases - especially in circumnavigation of the moon - necessary to take into account the effect of the sun and treat the problem as one involving four bodies.

The only way open is that of numerical integration of the differential equations of motion of the spacecraft. The potentialities of this method should not be overestimated, however. It is true that any degree of computational accuracy is attainable but, in general, it is only possible to make statements about the interval of time spanned by the numerical integration and no more. The single exception to this is when the flight path closes upon itself, that is when a periodic solution is found (cf. Stromgren's periodic orbits).

However, the fact that modern electronic computers are already capable of calculating the flight paths of spacecraft is brought out by the following two examples:

1) The American IBM 704 computer in Cambridge (Mass.) took 21 secs to calculate the orbital elements of Sputnik I from the observation data [26]. An unusually adept astronomer requires at least a 1,000 times as long, that is about six hours, to make the same calculations. The IBM 704 can carry out 40,000 calculating operations per second.

2) In a rocket launching there is usually two to three minutes between burn-out of the second stage and ignition of the third stage. In this short interval electronic computers have been successfully employed to derive preliminary flight path elements from the initial observation data available, so that this information can be used for triggering the ignition of the third stage in the sense of a desired or necessary flight path correction.

Of course, such rapid intervention presupposes that the programming has been worked out in advance.

Since 1957 a series of investigations and flight path calculations have already been made by means of numerical integration and these are discussed briefly below. With one exception these calculations are devoted to so-called passive flights, which are determined solely by the burn-out velocity.

The Soviet flight path calculations

The Russian investigations have been made primarily by W.A. Yegorov and his colleagues.

Yegorov has shown [27], [28], [29] that it is not even absolutely necessary to bring the spacecraft into a path leading directly to the moon. A path with its apogee about 250,000 km from the earth is sufficient; this is two thirds of the distance to the moon. The disturbing influence of the moon gradually modifies and broadens this path, until, finally, after hundreds of orbits, it extends as far as the moon. This would constitute a theoretical minimum flight path, which, however, could have no importance in practice (Fig. 11).

Yegorov then proceeded to calculate a very large number of possible flight paths to and around the moon, continually changing the starting conditions. His results may be summarized as follows:

a) Flight paths to and around the moon.

1) It is easy to calculate a flight path intended to hit the moon. If, for example, the actual moon flight path begins at burn-out 200 km above the earth, the moon will still be hit, if the error in burn-out velocity is not more than 0.3° in angular direction or 50 m/sec in speed.
2) It is also easy to calculate a flight path which circles the moon at a great distance.
3) However, it is very difficult to calculate a path which circles the moon at close range.

Circling the moon at close range, at a distance of about 27,000 km, requires an accuracy of 17" in the launching angle at burn-out (!). Today this still lies far beyond the accuracy of which we are technically capable.

b) Flight paths around the moon and the earth.

These flight paths, which bring the spacecraft back into the vicinity of the earth, are, of course, of the greatest practical interest. According to Yegorov they are improbably unstable and very sensitive to the slightest variation in velocity (a few mm/sec!).

A few examples of the numerous flight paths calculated by Yegorov are given in Fig. 12, assuming that the lunar orbit is circular.

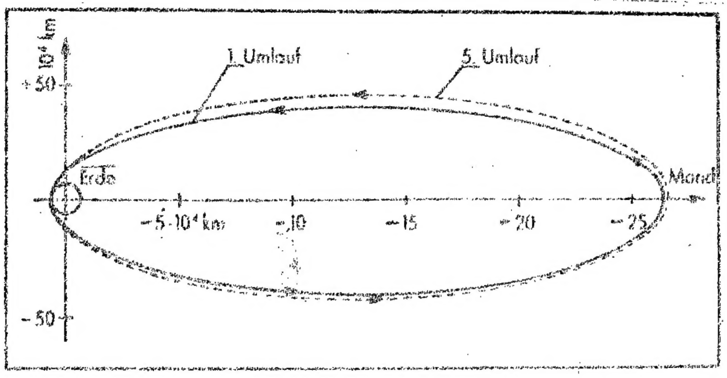

Fig. 11: Minimum flight path for reaching the moon according to Yegorov. (Umlauf = orbit; Mond = moon; Erde = earth). Figures are in units of 1,000 km. From O.W.Gail and W.Petri, Weltraumfahrt (Space Travel), publ. by Hanns Reich, Munich, 1958, Fig. 38, p. 92.

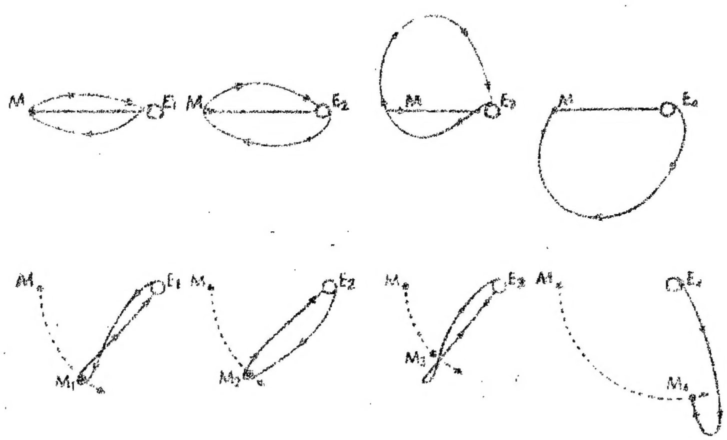

Fig. 12: A few examples of passive flight paths to the moon and back and a flight path hitting the moon. (According to W.A. Yegorov).

In the four examples illustrated the upper row in Fig. 12 shows the flight paths in a rotating coordinate system, related to the fixed line drawn between earth and moon (ME_1, ME_2, ME_3 and ME_4).

the row beneath reproduces the same motion in a stationary coordinate system, in which M, E_1 and so on denote the respective starting positions of earth and moon. Also shown are the positions M_1, M_2 ... of the moon, occupied by the latter when the spacecraft has traveled the earth-moon distance from the earth. In the third example it looks as if the moon is not circled. However, this is not so, for in the further course of the flight, in which the spacecraft goes beyond the lunar orbit, the moon naturally continues along its path and when the spacecraft re-crosses the lunar orbit on its return journey, it passes the moon on the other side. Thus, in this case too the moon is circled. In the fourth example, the moon is hit from behind. Unfortunately, Yegorov has only made a partial allowance for the effect of the sun in his calculations.

The American and the first German flight path calculations

The acuteness of the problem is indicated by the fact that American work has been published almost simultaneously with the Russian investigations.

a) Krafft A. Ehricke's flight path calculations.

Krafft A. Ehricke has published his investigations in a series of fundamental works, only a few of the most important conclusions of which can be given here [30], [31], [32].

In the gravitational field between earth and moon the velocity decreases more rapidly, anyhow, than in the two-body problem, for the gravitational vectors of earth and moon are mutually opposed. Thus, at times the sun, as a fourth body, will be capable of playing an important part. In his "Cislunar Orbits" [30], therefore, Krafft Ehricke investigates the effect of the sun with somewhat greater precision.

Naturally, compared with the original burn-out velocity of the moonship the absolute magnitude of the solar effect is small but it should not be overlooked that the effect of the sun extends over a fairly long interval of time and that in this section of the flight path the velocity of the moon probe itself has already become very much smaller. Thus, the sun may produce relatively large movements, i.e. flight path changes, which for their part lead to considerable seond-order disturbances due to the moon.

This can be seen very clearly in the worked out example shown in Figs. 13 and 14. The starting conditions are:

The moon probe starts at an altitude of 555.9 km (= 300 nautical miles) above the surface of the earth with a velocity of 10.6518 km/sec.

As at this altitude the escape velocity is 10.7314 km/sec, the launching velocity is 99.26% of this figure. The initial position is such that the moon lies at 50° below the direction to the sun. The lunar orbit is again assumed to be circular. The flight path of the moon probe is then calculated, once without and once with the effect of the sun, i.e. in the form of a three- and a four-body problem.

In the first case a minimum distance of 1708.5 km from the center of the moon is achieved; this means that the moon, the radius of which is 1,738.0 km, is hit. In the second case, in which the effect of the sun is taken into account, the same initial conditions give a minimum distance of 4,134.0 km from the center of the moon. The moon is then circled at a height of 2,336 km above its surface. This shows very clearly that we cannot calculate and ignore the effect of the sun, at any rate if we start very close to the escape velocity.

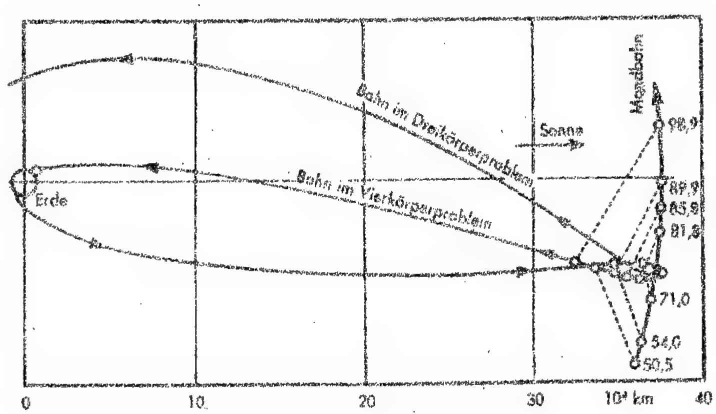

Fig. 13: A flight path to the moon and back disregarding and allowing for the effect of the sun. According to Krafft A. Ehricke: Instrumented Comets - Astronautics of solar and planetary probes; VIIIth International Astronautical Congress, Barcelona, 1957, p. 100. (Sonne = sun; Erde = earth; Mondbahn = lunar orbit; Bahn im Dreikorperproblem = path in the three-body problem; Bahn im Vierkorperproblem = path in the four-body problem).

The modification of the flight path will be much greater when the probe returns in the direction of the earth, however. Without taking the sun into account the craft will pass the earth at a distance of 37,000 to 55,000 km, whereas in the four-body problem it

reenters the earth's atmosphere. The sensitivity of the return path following an almost hyperbolic encounter with the moon is extraordinarily high (cf. Yegorov).

Fig. 13 shows the differences in the two paths following from the same starting conditions when treated as three- and four-body problems. Fig. 14 shows the sections of the flight paths close to the moon to an enlarged scale. The figures shown at the edge of the circular lunar orbit (Fig. 13) and the corresponding points in the path of the moon probe indicate the time in hours which has passed since the start of the flight.

This worked out example is particularly important as it represents the most important practical case: the other side of the moon is more or less fully illuminated, while we observe a new moon.

Ehricke has also worked out other examples and made an investigation of the question of accuracy, which we can do no more than refer to here [31], [32].

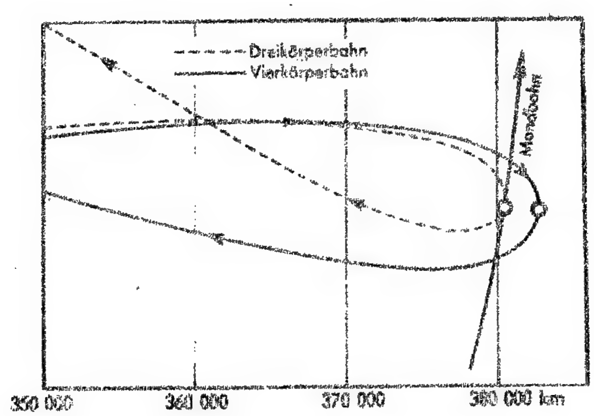

Fig. 14: Part of the flight path shown in Fig. 13 lying in the immediate vicinity of the moon. (---- three-body problem; —— four-body problem; Mondbahn = lunar orbit).

b) The investigations of M.W. Hunter, W.B. Klemperer, R.J. Gunkel and others.

The investigations of M.W. Hunter, W.B. Klemperer and R.J. Gunkel [33] have quite a different aim, namely a moon shot that will hit the

moon or even quite definite parts of the moon. Starting from the fact that a flight path to the moon is particularly sensitive to an error in direction at burn-out, a first - as far as the author knows - attempt is made to introduce active flight paths into the problem. These are flight paths which can be modified by means of an additional brief impulse during flight.

As we know, in the two-body problem the moon is reached in about 5.5 days with an elliptical initial velocity 1 % less than the escape velocity. The full escape velocity in a parabolic path requires only two days and if the escape velocity is exceeded by only 5 %, the duration of the flight along a hyperbolic path is already reduced to one day.

In order to get more precise data, flight path calculations have been carried out by numerical integration again with an IBM 704, initially for an idealized two-dimensional earth-moon model and a burn-out at 542 km above the surface of the earth. Detailed results are contained in papers by both H.A. Lieske [34] and G.C. Goldbaum and R.J. Gunkel [35].

It appears, first of all, that for a phase angle of 112.5° at the moment of starting, corresponding to a lunar position 22° below the eastern horizon at the launching site, there is a minimum-energy elliptical flight path (burn-out velocity = 10.6070 km/sec), which will hit the moon. The permissible error in the magnitude of the velocity is then only ± 1.524 m/sec, however. Nevertheless, if the initial velocity is just slightly increased to 10.6680 km/sec, the permissible error becomes ten times greater, i.e. 15.24 m/sec. Relationships are most favorable, if the flight path is begun at a hyperbolic initial velocity of 10.7442 km/sec (i.e. less than ½ % over the escape velocity); the maximum possible permissible error in the initial velocity is then ± 61 m/sec. If we add the angular errors, which are permissible if the moon is still to be hit, we get the following table:

Permissible errors in the initial velocity for a moon-hit

Initial velocity	Permissible Errors in velocity	Permissible Errors in launching angle
10.6070 km/sec	± 1.524 m/sec	± 1.7°
10.6680 km/sec	± 15.24 m/sec	± 17.2'
10.7442 km/sec	± 60.96 m/sec	± 10.3'

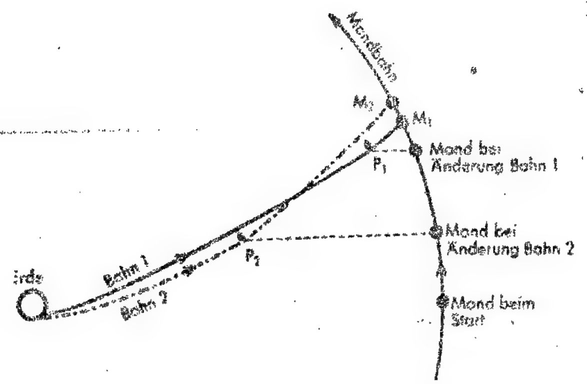

Fig. 15: Two active lunar shot flight paths. (According to M.W.Hunter, W.B.Klemperer, R.J.Gunkel: Impulsive mid-course correction of a lunar shot. Douglas Aircraft Company, Inc., Engrg. Paper No. 674, 1958) Mondbahn = lunar orbit; Mond bei Anderung Bahn 1,2 = moon at correction of flight path 1,2; Mond beim Start = moon at launching; Bahn 1,2 = flight path 1,2; Erde = earth.

At first this result appears surprising. In order to be more certain of hitting the lunar target, therefore, the authors propose to correct the flight path in mid-course by means of an additional impulse. To make this possible, they put forward a number of technical suggestions. Where the additional impulse is triggered, these active flight paths have a sudden kink, as the two examples shown in Fig. 15 illustrate. In the light of actual relationships a three-dimensional analysis was also carried out. This included the following factors, neglected in the two-dimensional calculations:

a) the inclination and eccentricity of the lunar orbit,
b) the geographical latitude and hour angle (longitude) of the launching site,
c) the gravitational fields of the sun and other planets.

Full details are given in the report by J.C. Walker [36]. In the three-dimensional problem it is worth noting that the angle between the plane of the moon rocket flight path and the plane of the lunar orbit has a considerable effect on the results.

c) The first German calculations.

The first German flight path calculations, made by B. Thuring [37], have also just been made available. They were carried out using the large Univac-Factronic computer installation in Frankfurt am Main.

B. Thuring calculated two special flight paths including the moon in the space around earth and moon and showed how the third (massless) body, i.e. the spacecraft, can, at a very low velocity, pass the "bottleneck", formed by the Hill boundary curve close to the libration point L_1 (or L_2). This investigation is an important supplement to the work of Yegorov; however, it would seem desirable to take into account disturbances due to the sun.

The first moon-launchings

In connection with a moon launching two further practical points deserve attention. If it is desired to shorten the flight path and the duration of the flight, then the moment of launching would be most favorably timed, if it were a few days before the lunar perigee, which occurs once a month, as the moon is then some 21,000 km nearer than when at its mean distance from the earth. On the other hand, if it is desired to observe the other side of the moon, which is permanently turned away from us, then the lunar rocket must reach the vicinity of the moon, when the other side is well illuminated. This is the case at new moon.

Of course, the complex motion of the moon means that these two conditions are not fulfilled simultaneously each month. A period does occur, however, at intervals of about 13½ months, when both perigee and new moon approximately coincide each month. This was so, for example, in the months from August to December 1958. For this reason the first four American moon launchings were planned for this period.

Details are given in the following brief review:

American moon-launching attempts August through December 1958.

Launching	Name	Launching date	New Moon	Perigee	Result
1	-	17 Aug.	15 Aug.	17 Aug.	exploded after 77 s
2	Pioneer I	11 Oct.	12 Oct.	13 Oct.	about 115,000 km alt.
3	Pioneer II	8 Nov.	11 Nov.	10 Nov.	3rd stage failed to ignite, 1,600 km alt.
4	Pioneer III	6 Dec.	10 Dec.	9 Dec.	about 107,500 km alt.

Pioneers I and III both covered about one third of the distance to the moon and then returned. This was due only to failure to achieve the required burn-out velocity. For Pioneer III, for example, a burn-out velocity of 10.729 km/sec had been computed; however, only 10.485 km/sec was achieved; i.e. just 250 m/sec too little. This confirms the above theoretical reasoning in connection with the maximum distance achieved and its sensitivity to a modification in the burn-out velocity. Pioneer I, moreover, deviated from its course by approximately 4° . Fig. 16 gives a rough picture of the flight path of Pioneer III, projected on a rotating meridian plane.

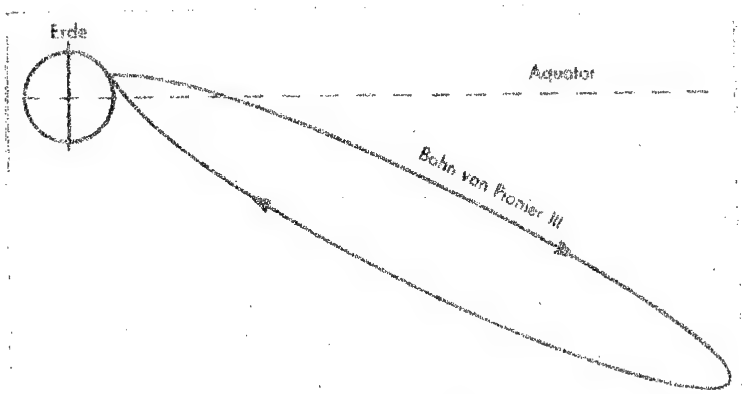

Fig. 16: The flight path of Pioneer III, projected on a rotating meridian plane. (According to James A. van Allen, Louis A. Frank, Nature, No. 4659, Feb. 1959, p. 432). Erde = earth; Aquator = equator; Bahn von Pionier III = flight path of Pioneer III.

Their unfortunate practical experiences with a launching velocity in the vicinity of the escape velocity and just slightly less than that required have prompted the Americans to adopt in their more recent experiments a burn-out velocity slightly hyperbolic with respect to the earth. Thus, even with a small error it could be confidently anticipated that the moon would be reached, insofar as the velocity did not fall below the critical limit of the escape velocity as a result of unavoidable inaccuracy.

Thus both the Russian moon-launching of 2 January 1959 (Metschta) and the American shot of 3 March 1959 (Pioneer IV) were successful in reaching lunar distances. Both probes then went into orbits around the sun, which are not dissimilar. If the orbital

inclination is disregarded, the Russian artificial planet even reached the vicinity of the orbit of Mars (cf. Fig. 1).

Both artificial planets are very small and their orbits are not known precisely enough. It is scarcely to be expected that we shall ever be successful in observing them again. Their orbital periods are 443 days (Metschta) and 403 days (Pioneer IV). If in their orbit they return to the starting point, the earth, of course, will be in another position.

Conclusions

The consequences derived from the celestial mechanics of the three- and four-body problems have been confirmed. There is only one way of calculating the flight path of a space rocket: numerical integration with an electronic computer. Flight path calculations made so far have permitted very interesting conclusions. However, we are still far from having discovered and mastered every possible path. It would be desirable to satandardize the flight paths, i.e. always to depart from a fixed initial altitude. Investigations of the errors involved must also be taken further. It would also be rewarding to study the Hill boundary curves for the given mass relationships. The author is also of the opinion that there must also be periodic solutions for the sun-earth-moon-rocket four-body problem. Finding them - perhaps in the same way as E. Stromgren found his periodic solutions for the limited three-body problem - would be an acceptable and important task for the future.

Summary

In Part II the author reports on completed orbital calculations and shows, with reference to the most recent material, how detailed has been the work of the Soviet astronomer, V.A. Yegorov, who has calculated about one thousand different orbits. The American, Krafft Ehricke, has explained the importance of allowing for the disturbing effect of the sun. All the calculations indicate how heavily the flight paths depend on starting conditions, an accuracy of 17" being required. The German-American Prof. Klemperer has shown that errors can be compensated by influencing the flight path actively with control rockets. The latest German work has been done by Prof. Thuring, who has determined two narrow path curves leading to the moon based on the points of libration near the moon. The soundness of Prof. Klemperer's proposals has meantime been proved by the Soviet 's hitting of the moon on 13 September 1959.

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(without claim to completeness)

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